Light-Cone Quark Model Analysis of Radially Excited Pseudoscalar and Vector Mesons

Daniel Arndt and Chueng-Ryong Ji

Department of Physics, North Carolina State University, Raleigh, N.C. 27695-8202

Abstract

We present a relativistic constituent quark model to analyze the mass spectrum and hadronic properties of radially excited u and d quark sector mesons. Using a simple Gaussian function as a trial wave function for the variational principle of a QCD motivated Hamiltonian, we obtain the mass spectrum consistent with the experimental data. To do the same for several observables such as decay constants and form factors, it seems necessary to include both Dirac and Pauli form factors on the level of constituent quarks. Taking into account the quark form factors, we thus present the generalized formulas for the rho meson decay constant and form factors as well as the $\pi\gamma$ transition form factor. We also predict several hadronic properties for the radially excited states.

12.39.Ki,13.40.Gp,13.40.Hq,14.40.-n

I. INTRODUCTION

Based on a detailed analysis of the meson mass spectrum by Godfrey and Isgur (GI) [1], the light-cone (LC) approach has been adopted to describe the pion decay constant [2], the charge form factor of the pion [2–5], the electromagnetic form factors of the ρ meson [6] and the radiative $\rho\pi$ and $\omega\pi$ transition form factors [3,4]. In this approach, Cardarelli *et al.* used the eigenfunctions of the effective $q\bar{q}$ GI Hamiltonian as orbital wave functions which consist of a truncated set of at least 38 harmonic oscillator (HO) basic states. While these wave functions reproduce the meson mass spectrum very well, it became necessary to introduce Dirac and Pauli form factors on the level of the constituent quarks (CQ) in order to reproduce hadronic meson properties. This has been done to calculate the charge form factor of the pion, the radiative $\rho\pi$ and $\omega\pi$ transition form factors [2,4] (both including Dirac and Pauli CQ form factors) and the electromagnetic form factors of the ρ meson [6] (Dirac CQ form factor only).

Recently [7], however, it was observed that both the masses and the hadronic properties of ground state pseudoscalar and vector mesons can fairly well be reproduced by taking just a single $1S$-state HO wave function given by

$$\phi_{1S}(\vec{k}^2) = \frac{1}{\pi^{3/4}\beta^{3/2}} \exp\left(-\frac{k^2}{2\beta^2}\right). \tag{1.1}$$

In these calculations, the choice of the Gaussian parameter β has been obtained by the variational principle for the QCD motivated Hamiltonian including the Coulomb plus confining potential. In this paper, we adopt this model [7] and extend it to the radially excited states for the u and d quark sector where the masses of u and d CQ can be taken equal, $m_u = m_d$. The necessity of CQ Dirac and Pauli form factors was also confirmed in our analysis of both ground state and excited states. We thus present the generalized formulas for the rho meson decay constant and form factors as well as the $\pi\gamma$ transition form factor taking into account both Dirac and Pauli CQ form factors. Following the ground state analysis, it may not be so unreasonable to take $2S$ and $3S$ HO wave functions given by

$$\phi_{2S}(\vec{k}^2) = \frac{1}{\sqrt{6}\pi^{3/4}\beta^{7/2}} \left(-3\beta^2 + 2k^2\right) \exp\left(-\frac{k^2}{2\beta^2}\right) \tag{1.2}$$

and

$$\phi_{3S}(\vec{k}^2) = \frac{1}{2\sqrt{30}\pi^{3/4}\beta^{11/2}} \left(15\beta^4 - 20\beta^2 k^2 + 4k^4\right) \exp\left(-\frac{k^2}{2\beta^2}\right) \tag{1.3}$$

for the first and second excited states, respectively. However, we tried the variations from these wave functions and obtained more justifications for using Eqs. (1.2) and (1.3), as we will present in the next section.

The LC formalism (for a recent review see [8]), first introduced by Dirac [9], presents a natural framework to include the relativistic effects which turned out to be crucial to describe the low-lying mesons [1]. Distinctive features of the LC time ($\tau = t + z/c$) quantization include the suppression of vacuum fluctuations [10] and the conversion of the dynamical problem from boost to rotation [11], which has the compact parameter space. Various

hadronic properties are calculated using the well established formulation in the Drell–Yan–West $q^+ = 0$ frame [12]. We also use the Melosh transformation [13] to assign proper quantum numbers J^{PC} to the mesons.

In our model, the meson state $|M\rangle$ is represented by

$$|M\rangle = \Psi_{Q\bar{Q}}^M |Q\bar{Q}\rangle, \tag{1.4}$$

where Q and $\bar{Q}$ are the effectively dressed quark and antiquark. The model wave function consists of the radial wave function ϕ_{nS}, the spin-orbit wave function $\mathcal{R}$ obtained by the Melosh-transformation and a Jacobi factor

$$\Psi_{Q\bar{Q}}^M = \Psi(x, \vec{k}_\perp, \lambda_q, \lambda_{\bar{q}}) = \sqrt{\frac{\partial k_n}{\partial x}} \mathcal{R}(x, \vec{k}_\perp, \lambda_q, \lambda_{\bar{q}}) \phi_{nS}(x, \vec{k}_\perp). \tag{1.5}$$

Here, the Jacobian of the variable transformation $\{k_n, \vec{k}_\perp\} \to \{x, \vec{k}_\perp\}$ is given by $\frac{\partial k_n}{\partial x} = \frac{M_0}{4x(1-x)}$ and the spin-orbit wave function for spin (S, S_z) is given by

$$\mathcal{R}_{S,S_z}(x, \vec{k}_\perp, \lambda_q, \lambda_{\bar{q}}) = \sum_{\lambda\lambda'} \left\langle \lambda_q \left| \mathcal{R}_M^\dagger(x, \vec{k}_\perp, m_q) \right| \lambda \right\rangle \left\langle \lambda_{\bar{q}} \left| \mathcal{R}_M^\dagger(1-x, -\vec{k}_\perp, m_{\bar{q}}) \right| \lambda' \right\rangle$$

$$\times \left\langle \frac{1}{2}\frac{1}{2}, \lambda\lambda' | SS_z \right\rangle, \tag{1.6}$$

where the Melosh rotation is given by

$$\mathcal{R}_M^\dagger(x, \vec{k}_\perp, m) = \frac{m + xM_0 - i\sigma(\hat{n} \times \hat{k})}{\sqrt{(m + xM_0)^2 + \vec{k}_\perp^2}} \tag{1.7}$$

with $\hat{n} = (0, 0, 1)$. Also, the wave function is normalized as

$$\sum_{\nu\bar{\nu}} \int d^3k \left| \Psi_{Q\bar{Q}}^M(x, \vec{k}_\perp, \nu, \bar{\nu}) \right|^2 = 1, \tag{1.8}$$

where $\vec{k} = (k_n, \vec{k}_\perp)$ and $k_n = (x - 1/2)M_0$.

We treat the meson mass according to the invariant meson mass scheme where the meson mass square M_0^2 for the case $m_q = m_{\bar{q}}$ is given by $M_0^2 = \frac{\vec{k}_\perp^2 + m_q^2}{x(1-x)}$ with m_q being the CQ mass. The intrinsic LC variables x and $\vec{k}_\perp$ are given by $x = k_q^+/P^+ = 1 - k_{\bar{q}}^+/P^+$ and $\vec{k}_\perp = \vec{k}_{q\perp} - x\vec{P}_\perp = -\vec{k}_{\bar{q}\perp} + (1-x)\vec{P}_\perp$. Here, the subscript $\perp$ indicates the component perpendicular to the LC quantization axis $\hat{n}$ and the $+$ component of a 4-vector $k = (k^0, \vec{k})$ is given by $k^+ = k^0 + \hat{n} \cdot \hat{k}$. The total 4-momentum of a meson in the frame of $\vec{P}_\perp = \vec{0}_\perp$ is therefore given by $P = (P^+, M^2/P^+, \vec{0}_\perp)$ and those of the quark and antiquark are given by $k_q = \left(xP^+, \frac{m_q^2 + \vec{k}_\perp^2}{xP^+}, \vec{k}_\perp \right)$ and $k_{\bar{q}} = \left((1-x)P^+, \frac{m_{\bar{q}}^2 + \vec{k}_\perp^2}{xP^+}, -\vec{k}_\perp \right)$, respectively.

The paper is organized as follows: In Sec. II, we fix our model parameters. First, in Sec. II A, we set up the QCD motivated effective Hamiltonian for the $q\bar{q}$ interaction. Using the wave function in Eq. (1.1) we determine the Gaussian parameter β from the variational principle. With a relativistic hyperfine interaction we then determine the best potential parameters and CQ mass m_q to fit the mass spectrum for the radially excited states of the

u and d quark sector as well as the pion decay constant f_π. In Sec. II B, we introduce the CQ form factors and fix the parameters associated with these form factors using the pion form factor F_π and the transition magnetic moments $\mu_{\rho\pi}$ and $\mu_{\omega\pi}$. In Sec. III, we present the generalized formulas taking into account the CQ form factors and predict the rho decay constant f_ρ, the rho electromagnetic form factors and the $\pi^0 \to \gamma^*\gamma$ transition form factor $F_{\pi\gamma}$. We also predict several hadronic observables involving radially excited states in this section. Summary and discussion follow in Sec. IV. In the Appendix, the details of the fitting procedure for the CQ structure parameters are presented.

II. FIXING THE MODEL PARAMETERS

A. CQ Mass and Potential Parameters

We start with the QCD motivated effective $q\bar{q}$ Hamiltonian as given by [7,1] for the description of the meson mass spectrum

$$H_{q\bar{q}}\left|\Psi_{nlm}^{SS_z}\right\rangle = (H_0 + V_{q\bar{q}})\left|\Psi_{nlm}^{SS_z}\right\rangle = M_{q\bar{q}}\left|\Psi_{nlm}^{SS_z}\right\rangle, \tag{2.1}$$

where $M_{q\bar{q}}$ is the mass of the meson, the free Hamiltonian is $H_0 = \sqrt{m_q^2 + \vec{k}^2} + \sqrt{m_{\bar{q}}^2 + \vec{k}^2}$, $|\Psi_{nlm}^{SS_z} >$ is the meson wave function as given in Eq. (1.5) and $V_{q\bar{q}}$ is the addition of the central potential V_0 and the hyperfine interaction V_{hyp} that we present in detail below.

As shown in [7], we use the variational principle for the ground state wave function in Eq. (1.1) to determine the value of the variational parameter β by satisfying

$$\frac{\partial \langle\phi_{1S}|(H_0 + V_0)|\phi_{1S}\rangle}{\partial\beta} = 0, \tag{2.2}$$

where V_0 is the interacting potential consisting of (a) Coulomb plus HO, and (b) Coulomb plus linear potential

$$V_0(r) = a + V_{\text{conf}}(r) - \frac{4\kappa}{3r}. \tag{2.3}$$

Here, $V_{\text{conf}}(r) = br^2$ [$V_{\text{conf}}(r) = br$] for HO [linear] confining potentials. Once the optimal parameter β for the ground state is determined, we use the same β for the first and second excited states. This ensures that all these states are orthogonal.

To distinguish the vector meson from the pseudoscalar meson, we include a hyperfine interaction [1,14]

$$V_{\text{hyp}}(r) = \sqrt{\frac{m_q m_{\bar{q}}}{E_q E_{\bar{q}}}} \frac{32\pi\kappa \vec{S}_q \cdot \vec{S}_{\bar{q}}}{9 m_q m_{\bar{q}}} \delta^3(\vec{r}) \sqrt{\frac{m_q m_{\bar{q}}}{E_q E_{\bar{q}}}}, \tag{2.4}$$

where $E_q = \sqrt{m_q^2 + \vec{k}^2}$, $E_{\bar{q}} = \sqrt{m_{\bar{q}}^2 + \vec{k}^2}$ and $\left\langle \vec{S}_q \cdot \vec{S}_{\bar{q}} \right\rangle = 1/4$ [$-3/4$] for vector [pseudoscalar] mesons. Since we are dealing with light mesons including radially excited states, this relativistic correction is essential. If we were to calculate the mass splitting ΔM_{nS} between pseudoscalar and vector mesons using the non-relativistic V_{hyp} given by

4

$$V_{\text{hyp}}(r) = \frac{2\vec{S}_q \cdot \vec{S}_{\bar{q}}}{3m_q m_{\bar{q}}} \nabla^2 V_{\text{Coul}} = \frac{32\pi\kappa\vec{S}_q \cdot \vec{S}_{\bar{q}}}{9m_q m_{\bar{q}}} \delta^3(\vec{r}) \tag{2.5}$$

then one gets

$$\Delta M_{1S} : \Delta M_{2S} : \Delta M_{3S} = 8 : 12 : 15, \tag{2.6}$$

where

$$\Delta M_{nS} = \left\langle \phi_{nS} \left| \frac{32\pi\kappa}{9m_q m_{\bar{q}}} \delta^3(\vec{r}) \right| \phi_{nS} \right\rangle. \tag{2.7}$$

This result clearly contradicts with the experimental values $\Delta M_{1S} = 0.630$ GeV, $\Delta M_{2S} \approx 0.100$ GeV and $\Delta M_{3S} \approx 0.100$ GeV.

To determine the potential parameters, we proceeded as follows. First, we chose the quark mass $m_q = m_{\bar{q}}$ as an input parameter assuming $m_u = m_d$. For both HO and linear potentials, we tried reasonable CQ masses in the range 0.150 GeV $\lesssim m_q \lesssim 0.300$ GeV. The potential parameters a, b and κ were chosen to provide an optimal fit for π, $\pi(2S)$, ρ and $\rho(2S)$. Then, we predicted the meson masses for $\pi(3S)$ and $\rho(3S)$. Our results are summarized in Table I. As one can see, the results for a wide range of the CQ mass m_q are within the experimental limits of the mass spectrum. Furthermore, the difference between the results of HO and linear potential is quite small once the best fit parameters are chosen. Note that the identity of the higher resonances is not completely clear yet [15]. While there are some experimental evidences for rho meson with mass 1700 ± 20 MeV, there is also a reported resonance at 2149 ± 17 MeV. In fact, $\rho(1700)$ might be a $3S$-hybrid mixture. Further consideration should be made to see if our simple model may or may not be suitable to reproduce these higher resonances.

We have also examined the Gaussian smearing function to weaken the singularity of the Delta function in the hyperfine interaction

$$\delta^3(\vec{r}) \longrightarrow \frac{\sigma^3}{\pi^{3/2}} \exp(-\sigma^2 r^2). \tag{2.8}$$

It turns out, however, that the smearing effect is negligible in our model calculations and the well known value of $\sigma = 1.8$ [1] did not change our results appreciably.

The radial wave function in Eq. (1.1) has been used successfully to approximate the ground state wave function in a couple of papers as mentioned above. In order to make sure if the first and second excited states can be well approximated with Eqs. (1.2) and (1.3), we varied the wave function of the first excited state to the mixed wave function

$$|\widetilde{2S}\rangle = f_2 |2S\rangle + f_3 |3S\rangle, \tag{2.9}$$

where $|2S\rangle$ and $|3S\rangle$ denote the second and third HO states. We determined the parameters f_2 and f_3 using the variational principle in Eq. (2.2) as $f_2 = 0.982$ and $f_3 = 0.190$, showing that the added term is much suppressed. For the second excited state we used the wave function

$$|\widetilde{3S}\rangle = f_3 |2S\rangle - f_2 |3S\rangle \tag{2.10}$$

5

which is orthogonal to Eq. (2.9). However, the calculated mass eigenvalues for these states deviate less than 2% from our values determined by using Eqs. (1.2) and (1.3). Therefore, we trust that our approximation of using Eqs. (1.2) and (1.3) is well justified.

While all the parameter sets summarized in Table I give a good agreement with the mass spectrum, the pion decay constant is rather sensitive to the choice of m_q. Following the LC approach of [16,17] for the pion decay constant given by

$$\left\langle 0 \left| \bar{q} \gamma^+ \gamma_5 q \right| \vec{P}, 00 \right\rangle \sqrt{2P^+} = iP^+ \sqrt{2} f_\pi, \tag{2.11}$$

one gets

$$f_\pi = \frac{\sqrt{6}}{(2\pi)^{3/2}} \int d^2 \vec{k}_\perp \int dx \sqrt{\frac{\partial k_n}{\partial x}} \frac{m_q}{\sqrt{m_q^2 + \vec{k}_\perp^2}} \phi_{1S}(x, \vec{k}_\perp). \tag{2.12}$$

As shown in Table I, the calculated value of 91.9 MeV for $m_q = 0.190$ GeV and the HO potential is in good agreement with the experimental value of 92.4 ± 0.25 MeV from [15]. We therefore from now on use the parametrization where $m_q = 0.190$ GeV and $\beta = 0.4957$ GeV. In our work, we don't need to introduce an arbitrary axial-vector coupling constant on the level of CQ [2].

In Fig. 1, we compare our central potential with those of other publications [1,14,7] in the interesting range of up to 2 fm. Our potential seems quite comparable to the other calculations.

B. CQ Form Factor Parameters

While the CQ were often treated as pointlike particles, there are hints in the literature [2–6] that the CQ may not be treated as pointlike particles. Especially, the Gerasimov sum-rule calculation [18] indicates that the CQ should be treated as extended particles. Thus, we introduced Dirac and Pauli form factors on the level of CQ. In fact, the Gaussian parameter $\beta = 0.4957$ GeV of our parametrization is somewhat larger than that of other calculations [7,16,17,14] where β is generally in the range $\beta \approx 0.3 \dots 0.4$ GeV. Therefore, it seems important to take into account these CQ form factors in order to obtain comparable results with the experimental data for the hadronic properties.

We substitute $j_\mu = e_q \gamma_\mu$ at a quark-photon coupling by the more general form

$$J_{q\mu} = F_D^{(q)}(Q^2) e_q \gamma_\mu + F_P^{(q)}(Q^2) \kappa_q i \sigma_{\mu\nu} \frac{q^\nu}{2m_q}, \tag{2.13}$$

where the Dirac and Pauli form factors on the level of CQ are normalized as $F_D^{(q)}(0) = F_P^{(q)}(0) = 1$. Here, e_q and κ_q are the CQ charge and anomalous magnetic moment, $Q^2 = -q^2$ is the 4-momentum transfer square and $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. For the Dirac and Pauli CQ form factors F_D and F_P, we adopt simple monopole and dipole forms

$$F_D^{(q)}(Q^2) = \frac{1}{1 + <r_D^{(q)^2}> Q^2/6} \quad \text{and} \quad F_P^{(q)}(Q^2) = \frac{1}{\left(1 + <r_P^{(q)^2}> Q^2/12\right)^2}, \tag{2.14}$$

6

as other authors [4] used. It turns out to be sufficient for our purpose to use this simple monopole and dipole form.

To fix the model parameters $< r_D^2 >$, $< r_P^2 >$, κ_u and κ_d, we fit our model to the available pion form factor data and to the experimental values of the transition magnetic moments $\mu_{\rho\pi}$ and $\mu_{\omega\pi}$. The details of the fitting procedure are presented in the Appendix.

Our determined values for the parameters $< r_D^2 >$, $< r_P^2 >$, κ_u and κ_d are shown in Table II. Note that these parameters are comparable to those determined in [4]. Also, we calculated the ratio $(e_u + \kappa_u)/(e_d + \kappa_d)$ which is predicted in [18] as -1.80 ± 0.02, which is comparable to our value of -1.94.

In Fig. 2, we show our calculation for F_π and compare it to the experimental data taken from [19]. We also compare our calculation to a simple Vector Meson Dominance (VMD) model where $F_\pi^{\mathrm{VMD}}(Q^2) = 1/(1 + Q^2/M_\rho^2)$. In Fig. 3, we show the radiative transition form factors $F_{\rho\pi}$ and $F_{\omega\pi}$ and the body form factors $H_D^{\rho\pi}$ and $H_P^{\rho\pi}$.

III. CALCULATION OF HADRONIC PROPERTIES

Having fixed all the parameters of our model in Sec. II, we now present our predictions of various hadronic properties of pseudoscalar and vector mesons including also the radially excited states.

A. ρ^0 Decay Constant f_ρ

From the definition [17]

$$< 0|J_{q_\mu}|\vec{P}, 1J_3 > \sqrt{2P^+} = \epsilon_\mu(J_3)M_\rho f_\rho, \tag{3.1}$$

we calculate the rho decay constant f_ρ including both Dirac and Pauli form factors on the level of CQ by taking J_{q_μ} as defined in Eq. (2.13). We obtain for $\rho^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$

$$f_\rho = \frac{1}{\sqrt{2}} \left([e_u - e_d]F_D^{(q)}(M_\rho^2)I_D + [\kappa_u - \kappa_d]F_P^{(q)}(M_\rho^2)I_P \right), \tag{3.2}$$

where

$$I_D = \frac{\sqrt{3}}{(2\pi)^{3/2}} \int d^2\vec{k}_\perp \int dx \frac{1}{x(1-x)} \frac{1}{\sqrt{M_0}} \left(m + \frac{2\vec{k}_\perp^2}{\lambda} \right) \phi_{1S}(x, \vec{k}_\perp) \tag{3.3}$$

and

$$I_P = \frac{\sqrt{3}}{4(2\pi)^{3/2}m_q} \int d^2\vec{k}_\perp \int dx \frac{\sqrt{M_0}}{x(1-x)} \left(\frac{4\vec{k}_\perp^2}{\lambda} - M_0 \right) \phi_{1S}(x, \vec{k}_\perp). \tag{3.4}$$

Note here that the momentum transfer square equals the mass of the rho meson $Q^2 = M_\rho^2 = (0.770 \text{ GeV})^2$. While I_D has been already derived in [17], I_P is our new body form factor related to the Pauli form factor in the electromagnetic current operator J_{q_μ}. Using our parametrization (Table II) we get the value $f_\rho = 153$ MeV which is in a good agreement with the experimental data $f_\rho = 152.8 \pm 3.6$ MeV obtained from the width $\Gamma(\rho \to e^+e^-)$ [15].

B. ρ^+ Form Factors

Our calculation for the ρ^+ form factors follows the one presented in [20]. However, we again treat the CQ as extended objects characterized by Dirac and Pauli form factors in contrast to a treatment as point like ones.

In the standard LC frame the charge, magnetic and quadrupole form factors of a meson can be obtained from the plus component of three helicity matrix elements [21]

$$F_C = \frac{1}{(2\alpha + 1)} \left[\frac{16}{3} \alpha \frac{F_{+0}^+}{\sqrt{2\alpha}} - \frac{1}{3}(2\alpha - 3)F_{00}^+ + \frac{2}{3}(2\alpha - 1)F_{+-}^+ \right] \tag{3.5}$$

$$F_M = \frac{2}{(2\alpha + 1)} \left[(2\alpha - 1)\frac{F_{+0}^+}{\sqrt{2\alpha}} + F_{00}^+ - F_{+-}^+ \right] \tag{3.6}$$

$$F_Q = \frac{1}{(2\alpha + 1)} \left[2\frac{F_{+0}^+}{\sqrt{2\alpha}} - F_{00}^+ - \frac{\alpha + 1}{\alpha} F_{+-}^+ \right] \tag{3.7}$$

where $F_{\lambda'\lambda}^\mu = \langle P', \lambda' | (J_u^\mu - J_d^\mu) | P, \lambda \rangle$, $\alpha = \frac{Q^2}{4M_\rho^2}$ is a kinematic factor and J_q^μ is defined in Eq. (2.13). This representation is not unique; there are different prescriptions in the literature, as discussed in [6]. This ambiguity is reflected in the fact that the angular condition

$$\Delta(Q^2) = (1 + 2\alpha)F_{++}^+ + F_{+-}^+ - \sqrt{8\alpha}F_{+0}^+ - F_{00}^+ = 0 \tag{3.8}$$

is in general violated unless the exact Poincaré covariant current operator beyond one-body sector is used.

At zero momentum transfer, these form factors are proportional to the meson charge e, magnetic moment μ_1 and quadrupole moment Q_1:

$$F_C(0) = 1, \qquad eF_M(0) = 2M_\rho\mu_1 \qquad \text{and} \qquad eF_Q(0) = M_\rho^2 Q_1. \tag{3.9}$$

In the LC quark model, the matrix element $\langle P', \lambda' | J_q^\mu | P, \lambda \rangle$ can be calculated by the convolution of initial and final LC wave function of a meson

$$\langle P', \lambda' | J_q^\mu | P, \lambda \rangle = \sum_{\lambda_q, \lambda_{q'}, \lambda_{\bar{q}}} \int d^2\vec{k}_\perp \int dx \sqrt{\frac{\partial k_n}{\partial x}} \sqrt{\frac{\partial k_n'}{\partial x}} \phi_{1S}(x, \vec{k}_\perp) \phi_{1S}^*(x, \vec{k}_\perp')$$

$$\times \left[\frac{\bar{u}(k_{q'}, \lambda_{q'})}{\sqrt{k_{q'}^+}} \frac{J_q^\mu}{2} \frac{u(k_q, \lambda_q)}{\sqrt{k_q^+}} \right] \mathcal{R}'^*(\lambda_{q'}, \lambda_{\bar{q}}; 1, \lambda') \mathcal{R}(\lambda_q, \lambda_{\bar{q}}; 1, \lambda). \tag{3.10}$$

After a straightforward calculation, choosing the $+$ component of the current, we get

$$F_{\lambda'\lambda}^+(Q^2) = (e_u - e_d)F_D^{(q)}(Q^2)I_D^{\lambda'\lambda}(Q^2) + (\kappa_u - \kappa_d)F_P^{(q)}(Q^2)I_P^{\lambda'\lambda}(Q^2), \tag{3.11}$$

where the body form factors related to the Dirac part are given by

$$I_D^{00}(Q^2) = \int d^2\vec{k}_\perp \int dx \sqrt{\frac{\partial k_n}{\partial x}} \sqrt{\frac{\partial k_n'}{\partial x}} \phi_{1S}(x, \vec{k}_\perp) \phi_{1S}^*(x, \vec{k}_\perp') \frac{1}{x(1 - x)\lambda\lambda'}$$

$$\times \left[(1 - 2x)^2 \vec{k}_\perp \vec{k}_\perp' + (2x[1 - x]M_0' + m)(2x[1 - x]M_0 + m) \right], \tag{3.12}$$

8

$$I_D^{+0}(Q^2) = \int d^2\vec{k}_\perp \int dx \sqrt{\frac{\partial k_n}{\partial x}} \sqrt{\frac{\partial k_n'}{\partial x}} \phi_{1S}(x, \vec{k}_\perp) \phi_{1S}^*(x, \vec{k}_\perp') \frac{1-2x}{\sqrt{2} M_0' x \lambda \lambda'}$$
$$\times \left[2x k_x M_0'(M_0' - M_0) + Q \left(2k_y^2 - 2x(1-x)M_0' M_0 - M_0' m \right) \right], \qquad (3.13)$$

$$I_D^{+-}(Q^2) = \int d^2\vec{k}_\perp \int dx \sqrt{\frac{\partial k_n}{\partial x}} \sqrt{\frac{\partial k_n'}{\partial x}} \phi_{1S}(x, \vec{k}_\perp) \phi_{1S}^*(x, \vec{k}_\perp') \frac{1}{M_0 M_0' x(1-x) \lambda \lambda'}$$
$$\times \left[\left(mM_0 + mM_0' + 2m^2 + 2x(1-x)M_0 M_0' \right) k_x k_x' + \left(2m^2 - 2x[1-x]M_0 M_0' \right) k_y^2 \right.$$
$$\left. -2k_x^2 k_x'^2 - m \left(\lambda' k_x^2 + \lambda k_x'^2 \right) + 2k_y^4 \right] \qquad (3.14)$$

and

$$I_D^{++}(Q^2) = \int d^2\vec{k}_\perp \int dx \sqrt{\frac{\partial k_n}{\partial x}} \sqrt{\frac{\partial k_n'}{\partial x}} \phi_{1S}(x, \vec{k}_\perp) \phi_{1S}^*(x, \vec{k}_\perp') \frac{1}{M_0 M_0' x(1-x) \lambda \lambda'}$$
$$\times \left[\left(k_\perp'^2 + m\lambda' \right) \left(k_\perp^2 + m\lambda \right) + \left(\vec{k}_\perp' \vec{k}_\perp \right)^2 - (1-x)^2 Q^2 k_y^2 \right.$$
$$\left. + \left(2m^2 + mM_0 + mM_0' + [2x^2 - 2x + 1]M_0 M_0' \right) \vec{k}_\perp' \vec{k}_\perp \right]. \qquad (3.15)$$

Here, $\lambda' = 2m + M_0'$. These formulas have already been used by the authors of [6] where they calculated the ρ^+ form factors including only the Dirac part of the CQ form factors by assuming that the anomalous magnetic moments of the CQ are negligible. We, however, present a complete calculation including also the Pauli form factor part. For the new body form factors related to the Pauli part, we obtain

$$I_P^{00}(Q^2) = \int d^2\vec{k}_\perp \int dx \sqrt{\frac{\partial k_n}{\partial x}} \sqrt{\frac{\partial k_n'}{\partial x}} \phi_{1S}(x, \vec{k}_\perp) \phi_{1S}^*(x, \vec{k}_\perp') \frac{Q(1-2x)}{2x(1-x)m\lambda\lambda'}$$
$$\times \left[\frac{m\lambda' + 2k_\perp'^2}{M_0'} k_x - \frac{m\lambda + 2k_\perp^2}{M_0} k_x' \right], \qquad (3.16)$$

$$I_P^{+0}(Q^2) = \int d^2\vec{k}_\perp \int dx \sqrt{\frac{\partial k_n}{\partial x}} \sqrt{\frac{\partial k_n'}{\partial x}} \phi_{1S}(x, \vec{k}_\perp) \phi_{1S}^*(x, \vec{k}_\perp') \frac{Q}{2\sqrt{2}x(1-x)m\lambda\lambda'}$$
$$\times \left[\frac{1-2x}{M_0'} \left([m + xM_0']\vec{k}_\perp' \vec{k}_\perp - [m + (1-x)M_0'][k_x' k_x - k_y^2] \right) \right.$$
$$\left. -\frac{1}{M_0' M_0} \left([\lambda m + 2k_\perp^2][k_x'^2 - k_y^2] + [\lambda' m + k_\perp'^2][\lambda m + 2k_\perp^2] \right) \right], \qquad (3.17)$$

$$I_P^{+-}(Q^2) = \int d^2\vec{k}_\perp \int dx \sqrt{\frac{\partial k_n}{\partial x}} \sqrt{\frac{\partial k_n'}{\partial x}} \phi_{1S}(x, \vec{k}_\perp) \phi_{1S}^*(x, \vec{k}_\perp') \frac{Q}{2x(1-x)m\lambda\lambda' M_0 M_0'}$$
$$\times \left[(m + [1-x]M_0') \left(k_x' \left[k_x^2 - k_y^2 \right] - 2k_x k_y^2 \right) + \left(k_\perp'^2 + \lambda' m \right)(m + xM_0)k_x \right.$$
$$\left. -(m + [1-x]M_0) \left(k_x \left[k_x'^2 - k_y^2 \right] - 2k_x' k_y^2 \right) - \left(k_\perp^2 + \lambda m \right)(m + xM_0')k_x' \right] \qquad (3.18)$$

and

$$I_P^{++}(Q^2) = \int d^2\vec{k}_\perp \int dx \sqrt{\frac{\partial k_n}{\partial x}} \sqrt{\frac{\partial k_n'}{\partial x}} \phi_{1S}(x,\vec{k}_\perp) \phi_{1S}^*(x,\vec{k}_\perp') \frac{Q}{2x(1-x)m\lambda\lambda' M_0 M_0'}$$

$$\times \left[-(m + (1-x)M_0') \left(k_\perp^2 + \lambda m\right) k_x' - (m + xM_0) \left(k_x \left[k_x'^2 - k_y^2\right] + 2k_x' k_y^2\right)\right.$$

$$\left. + (m + (1-x)M_0) \left(k_\perp'^2 + \lambda' m\right) k_x + (m + xM_0') \left(k_x' \left[k_x^2 - k_y^2\right] + 2k_x k_y^2\right)\right]. \quad (3.19)$$

We show the result for the ρ^+ form factors using our parametrization (Table II) in Fig. 4. In Table III, we list our calculations for μ_1 and Q_1. Note that our result is quite comparable to other calculations [22,6,20].

Using Eq. (3.8), we have also calculated the violation of the angular condition and compared it in Fig. 5 with several other calculations [6,20]. The violation of the angular condition seems to be suppressed by taking into account the structure of CQ, compared with the other calculations.

C. $\pi^0 \to \gamma^* \gamma$ Transition Form Factor

The $\pi^0 \to \gamma^* \gamma$ form factor $F_{\pi\gamma}$ has been calculated quite successfully in several models where the CQ has been treated as a pointlike particle [17,7]. We show the generalized formulation including both Dirac and Pauli form factor of the CQ.

The $\pi^0 \to \gamma^* \gamma$ transition form factor in leading order is defined as

$$\Gamma_{q_\mu} = iG_q(Q^2)\epsilon_{\mu\nu\rho\sigma} P^\nu \epsilon^\rho q^{*\sigma}, \quad (3.20)$$

where P is the momentum of the incident pion and q^* is the momentum of the virtual photon. Taking $P^+ = 1$ the vertex factor is given by

$$\Gamma_q^+ = \frac{\sqrt{3}}{4(2\pi)^{3/2}} \sum_{\lambda,\lambda',\bar{\lambda}} \int d^2\vec{k}_\perp \int dx_1 dx_2 \delta(1 - x_1 - x_2) \sqrt{\frac{\partial k_n}{\partial x}} \phi_{1S}(x,\vec{k}_\perp)$$

$$\times \left[\mathcal{R}(\lambda,\bar{\lambda};0,0) \frac{\bar{v}_{\bar{\lambda}}(x_2, -\vec{k}_\perp)}{\sqrt{x_2}} \epsilon_\mu J_q^\mu \frac{u_{\lambda'}(x_1, \vec{k}_\perp + \vec{q}_\perp)}{\sqrt{x_1}} \frac{\bar{u}_{\lambda'}(x_1, \vec{k}_\perp + \vec{q}_\perp)}{\sqrt{x_1}} J_q^+ \frac{u_\lambda(x_1, \vec{k}_\perp)}{\sqrt{x_1}} \right.$$

$$\left. \frac{1}{q_\perp^2 - \frac{(\vec{k}_\perp + \vec{q}_\perp)^2 + m^2}{x_1} - \frac{k_\perp^2 + m^2}{x_2}} + (x_1 \leftrightarrow x_2) \right]. \quad (3.21)$$

Note that we have to plug in J_{q_μ} as defined in (2.13) at the vertices of the real and the virtual photon. Here, of course, the momentum transfer square for the real photon should be zero. After a straightforward calculation, we get

$$F_{\pi\gamma}(Q^2) = G_u(Q^2) - G_d(Q^2), \quad (3.22)$$

where

10

$$G_q(Q^2) = \frac{\sqrt{3}}{\sqrt{2}Q(2\pi)^{3/2}} \int d^2\vec{k}_\perp \int dx \frac{\sqrt{M_0}}{M_0'^2 x^2(1-x)}\phi_{1S}(x,\vec{k}_\perp)$$
$$\times \left[e_q F_D(Q^2)\frac{2}{M_0}\left(mQe_q - \frac{\kappa_q}{2m}\left[(k_x + 2(1-x)Q)\,xM_0^2 + (1-x)k_x Q^2\right]\right)\right.$$
$$+\kappa_q F_P(Q^2)\frac{1}{m(1-x)M_0}\left(e_q Q\left[xk_\perp^2 + m^2\right] + e_q[1-x]Q\left[k_x^2 - k_y^2\right]\right.$$
$$\left.\left. +e_q[1-x]Q^2 k_x e_q - \kappa_q\frac{Q}{2}\left[k_\perp^2 + m^2 - (1-x)^2Q^2\right]\right)\right]. \tag{3.23}$$

Our result is shown in Fig. 6. As Jaus [17] pointed out, the hadron structure of the neutral pseudoscalar meson (π^0) may not be well enough approximated by the one-loop calculation. Gluon-exchange effects may introduce additional structure that might lead to a mechanism analogous to the flavor mixing of isoscalar states. Especially, at $Q^2 = 0$, from the one-loop formula given by Eqs. (3.22) and (3.23) and the definition of $\Gamma_{\pi\gamma}$ given by

$$\Gamma_{\pi\gamma} = \frac{\pi}{4}\alpha^2 F_{\pi\gamma}(0)^2 M_\pi^3, \tag{3.24}$$

we obtaion $\Gamma_{\pi\gamma} = 4.97$ eV. However, the agreement with the experimental data $\Gamma_{\pi\gamma} = 7.8 \pm 0.5$ eV can be obtained by taking into account the PCAC and the anomaly of the axial-vector current [23,24], which predicts

$$F_{\pi\gamma}(0) = \frac{1}{4\pi^2 f_\pi}. \tag{3.25}$$

Using this and Eq. (3.24), we obtain $\Gamma_{\pi\gamma} = 7.82$ eV.

On the other hand, the high-momentum transfer region ($Q^2 \gtrsim$ few GeV^2) is dominated by the one-loop off-shell quark contribution as evidenced from the Q^2-behaviour $\left(\sim \frac{1}{Q^2}\right)$. Recently [25], it has been shown that for high momentum transfer region the transition form factor can be obtained by the renormalization scale and scheme independent PCQD calculation as

$$F_{\pi\gamma}(Q^2) = \frac{2f_\pi}{Q^2}\left(1 - \frac{5}{3}\frac{\alpha_V(e^{-3/2}Q)}{\pi}\right), \tag{3.26}$$

where $\alpha_V(e^{-3/2}Q)/\pi \approx 0.12$. Using this relation, one can get a good agreement with the experimental data for $Q^2 \gtrsim 2$ GeV^2, as shown in Fig. 6.

D. Predictions for Radially Excited States

We also calculate several hadronic properties of radially excited states using the same parameters shown in Table II. For these calculations, we use the radial wave functions ϕ_{2S} and ϕ_{3S} from Eqs. (1.2) and (1.3), respectively.

In Table IV, we summarized our results including the transition magnetic moments $\mu_{\rho'^+\pi^+}$ and $\mu_{\omega'\pi^0}$, which are the transitions from the first exited state to the ground state. While the pion and rho decay constants are proportional to the wave functions at the origin for the ground state, the nodal structures of $2S$ and $3S$ radially excited states make intuitive predictions on the ordering of decay constant magnitudes untenable. We also show our result for the pion form factors for the first and second excited states in Fig. 7.

11

IV. SUMMARY AND DISCUSSION

In this paper, we used a simple relativistic CQ model to calculate various properties of radially excited u and d quark sector mesons. We find that a QCD motivated Hamiltonian with a relativistically corrected hyperfine interaction yield the mass spectrum comparable with data. We also treated the CQ not as pointlike objects but as extended ones, thereby introducing both Dirac and Pauli electromagnetic form factors for the CQ, and were able to obtain a reasonable agreement with the data for hadronic properties like the pion decay constant f_π, the pion form factor F_π and the transition magnetic moments $\mu_{\rho\pi}$ and $\mu_{\omega\pi}$. Furthermore, we used this model to calculate the rho decay constant f_ρ, the rho form factors and the $\pi^0 \to \gamma^*\gamma$ transition form factor $F_{\pi\gamma}$, presenting for the first time generalized formulas including both Dirac and Pauli form factors on the level of the CQ. We find that the angular condition $\Delta(Q^2)$ is better satisfied when the CQ form factors are included. However, our model is intrinsically limited to small momentum transfer range. As an application of our model, we also calculated several hadronic properties for radially excited states. Further experimental data on radially excited states will give more stringent test of our model.

ACKNOWLEDGEMENT

APPENDIX: DETAILS OF THE DETERMINATION OF THE CQ PARAMETERS

The calculation of F_π has been considered in a number of references [26,27,2,4]. For $\pi^+ = u\bar{d}$ one gets

$$F_\pi(Q^2) = \left(e_u F_D^{(u)}(Q^2) + e_{\bar{d}} F_D^{(\bar{d})}(Q^2)\right) H_D^\pi(Q^2) + \left(\kappa_u F_P^{(u)}(Q^2) + \kappa_{\bar{d}} F_P^{(\bar{d})}(Q^2)\right) H_P^\pi(Q^2)$$

$$= F_D^{(q)}(Q^2) H_D^\pi + (\kappa_u - \kappa_d) F_P^{(q)}(Q^2) H_P^\pi(Q^2), \tag{A1}$$

assuming the same internal structure for the u and d CQ, $F_D^{(u)} = F_D^{(d)} \equiv F_D^{(q)}$ and $F_P^{(u)} = F_P^{(d)} \equiv F_P^{(q)}$. The body form factors H_D^π and H_P^π are given by [4]

$$H_D^\pi(Q^2) = \int d^2\vec{k}_\perp \int dx \sqrt{\frac{\partial k_n}{\partial x}} \sqrt{\frac{\partial k_n'}{\partial x}} \phi_{1S}(x, \vec{k}_\perp) \phi_{1S}^*(x, \vec{k}_\perp') \frac{m^2 + \vec{k}_\perp \vec{k}_\perp'}{x(1-x)M_0 M_0'} \tag{A2}$$

and

$$H_P^\pi(Q^2) = \int d^2\vec{k}_\perp \int dx \sqrt{\frac{\partial k_n}{\partial x}} \sqrt{\frac{\partial k_n'}{\partial x}} \phi_{1S}(x, \vec{k}_\perp) \phi_{1S}^*(x, \vec{k}_\perp') \frac{-Q^2}{2x M_0 M_0'}. \tag{A3}$$

M_0' is given by $M_0'^2 = \frac{\vec{k}_\perp'^2 + m_q^2}{x(1-x)}$ and $\vec{k}_\perp'$ by $\vec{k}_\perp' = \vec{k}_\perp + (1-x)\vec{q}_\perp$.

Following a similar spin and flavour algebra for $\rho^+ = u\bar{d}$, $\omega = 1/\sqrt{2}(u\bar{u} + d\bar{d})$ and $\pi^0 = 1/\sqrt{2}(u\bar{u} - d\bar{d})$, one gets for the radiative $\rho^+ \rightarrow \pi^+ \gamma^*$ and $\omega \rightarrow \pi^0 \gamma^*$ transition form factors

$$F_{\rho\pi}(Q^2) = \left(e_u F_D^{(u)}(Q^2) - e_{\bar{d}} F_D^{(\bar{d})}(Q^2)\right) H_D^{\rho\pi}(Q^2) + \left(\kappa_u F_P^{(u)}(Q^2) - \kappa_{\bar{d}} F_P^{(\bar{d})}(Q^2)\right) H_P^{\rho\pi}(Q^2)$$

$$= \frac{1}{3} F_D^{(q)} H_D^{\rho\pi}(Q^2) + (\kappa_u + \kappa_d) F_P^{(q)}(Q^2) H_P^{\rho\pi}(Q^2) \tag{A4}$$

and

$$F_{\omega\pi}(Q^2) = \left(e_u F_D^{(u)}(Q^2) + e_{\bar{d}} F_D^{(\bar{d})}(Q^2)\right) H_D^{\rho\pi}(Q^2) + \left(\kappa_u F_P^{(u)}(Q^2) + \kappa_{\bar{d}} F_P^{(\bar{d})}(Q^2)\right) H_P^{\rho\pi}(Q^2)$$

$$= F_D^{(q)} H_D^{\rho\pi}(Q^2) + (\kappa_u - \kappa_d) F_P^{(q)}(Q^2) H_P^{\rho\pi}(Q^2), \tag{A5}$$

where the body form factors $H_D^{\rho\pi}$ and $H_P^{\omega\pi}$ are given by [4]

$$H_D^{\rho\pi}(Q^2) = 2 \int d^2\vec{k}_\perp \int dx \sqrt{\frac{\partial k_n}{\partial x}} \sqrt{\frac{\partial k'_n}{\partial x}} \phi_{1S}(x, \vec{k}_\perp) \phi_{1S}^*(x, \vec{k}'_\perp) \frac{m\lambda + 2k_y^2}{x M_0 M'_0 \lambda} \tag{A6}$$

and

$$H_P^{\rho\pi}(Q^2) = \int d^2\vec{k}_\perp \int dx \sqrt{\frac{\partial k_n}{\partial x}} \sqrt{\frac{\partial k'_n}{\partial x}} \phi_{1S}(x, \vec{k}_\perp) \phi_{1S}^*(x, \vec{k}'_\perp) \frac{\lambda\left(m^2 + \vec{k}_\perp \vec{k}'_\perp\right) - 2M_0(1-x)k_y^2}{x(1-x) M_0 M'_0 m \lambda}. \tag{A7}$$

Here, $\lambda = 2m + M_0$.

We are now able to fix the parameters κ_u and κ_d. The value of the transition form factors at $Q^2 = 0$, the transition magnetic moments $\mu_{\rho\pi}$ and $\mu_{\omega\pi}$, have been experimentally determined from the radiative decay widths of ρ and ω mesons [17,20] viz.

$$\Gamma(\rho \rightarrow \pi\gamma) = \frac{1}{3}\alpha F_{\rho\pi}(0)^2 \left(\frac{M_\rho^2 - M_\pi^2}{2M_\rho}\right)^3 \tag{A8}$$

and a similar expression for $\Gamma(\omega \rightarrow \pi\gamma)$ as $\mu_{\rho\pi} = 0.741 \pm 0.038$ GeV^{-1} and $\mu_{\omega\pi} = 2.33 \pm 0.06$ GeV^{-1}, respectively [15]. From (A4) and (A5) we get

$$\mu_{\rho\pi} = \frac{H_D^{\rho\pi}(0)}{3} + (\kappa_u + \kappa_d) H_P^{\rho\pi}(0) \tag{A9}$$

and

$$\mu_{\omega\pi} = H_D^{\omega\pi}(0) + (\kappa_u - \kappa_d) H_P^{\omega\pi}(0). \tag{A10}$$

By fitting the experimental value for $\mu_{\rho\pi}$ and $\mu_{\omega\pi}$ one gets $\kappa_u - \kappa_d = 0.138 \pm 0.015$ and $\kappa_u + \kappa_d = 0.036 \pm 0.01$ or, correspondingly, $\kappa_u = 0.087 \pm 0.013$ and $\kappa_d = -0.051 \pm 0.013$.

The mean square radii (MSR) associated with $F_D^{(q)}$ and $F_P^{(q)}$ are $<r_D^{(q)2}> = -6 \left.\frac{dF_D^{(q)}}{dQ^2}\right|_{Q^2=0}$ and $<r_P^{(q)2}> = -6 \left.\frac{dF_P^{(q)}}{dQ^2}\right|_{Q^2=0}$. Assuming the same internal electromagnetic structure for quark and antiquark, we have $r_D^{(u)} = r_D^{(d)} \equiv r_D^{(q)}$ and $r_P^{(u)} = r_P^{(d)} \equiv r_P^{(q)}$.

To determine $r_D^{(q)}$ and $r_P^{(q)}$, we fit our model to the well known electromagnetic form factor for the pion F_π, following the procedure outlined in [4]. Using (A1), we get

$$< r_\pi^2 > = -6 \left. \frac{dF_\pi}{dQ^2} \right|_{Q^2=0} = < r_D^2 > + < r_{H_D^\pi}^2 > + (\kappa_u - \kappa_d) < r_{H_P^\pi}^2 >, \quad \text{(A11)}$$

where $< r_{H_D^\pi}^2 > = -6 \left. \frac{dH_D^\pi}{dQ^2} \right|_{Q^2=0}$ and $< r_{H_P^\pi}^2 > = -6 \left. \frac{dH_P^\pi}{dQ^2} \right|_{Q^2=0}$ are the MSR associated with the body form factors H_D^π and H_D^π. Using our values for κ_u and κ_d, we find $< r_D^2 > = 0.0495 \pm 0.022$ fm² to obtain the experimental value $< r_\pi^2 > = 0.432 \pm 0.016$ fm² [28]. The last parameter to be determined, $< r_P^2 >$, is not affected by the $Q^2 \approx 0$ range of F_π (see Eq. (A11)) and can be chosen to be $< r_P^2 > = 0.136$ fm² to get a good agreement in the whole range of existing pion form factor data.

Finally, making use of the uncertainties due to the experimental errors, we determine our parametrization (see Table II) to get the best possible fit for the rho decay constant f_ρ.

REFERENCES

[1] S. Godfrey and N. Isgur, Phys. Rev. **D32**, 189 (1985).

[2] F. Cardarelli *et al.*, Phys. Lett. **B332**, 1 (1994).

[3] F. Cardarelli *et al.*, Few-Body Systems Suppl. **9**, 267 (1995).

[4] F. Cardarelli *et al.*, Phys. Lett. **B359**, 1 (1995).

[5] F. Cardarelli *et al.*, Phys. Rev. **D53**, 6682 (1996).

[6] F. Cardarelli *et al.*, Phys. Lett. **B349**, 393 (1995).

[7] H.-M. Choi and C.-R. Ji, Phys. Rev. **D59**, 074015 (1999).

[8] S. J. Brodsky, H.-C. Pauli, and S. S. Pinsky, Phys. Rept. **301**, 299 (1998).

[9] P. M. A. Dirac, Rev. Mod. Phys. **21**, 392 (1949).

[10] C.-R. Ji and S.-J. Rey, Phys. Rev. **D53**, 5815 (1996).

[11] C.-R. Ji and Y. Surya, Phys. Rev. **D46**, 3565 (1992).

[12] S. D. Drell and T.-M. Yan, Phys. Rev. Lett. **24**, 181 (1970).

[13] H. J. Melosh, Phys. Rev. **D9**, 1095 (1974).

[14] D. Scora and N. Isgur, Phys. Rev. **D52**, 2783 (1995).

[15] C. Caso *et al.*, Eur. Phys. J. **C3**, 1 (1998).

[16] W. Jaus, Phys. Rev. **D41**, 3394 (1990).

[17] W. Jaus, Phys. Rev. **D44**, 2851 (1991).

[18] S. B. Gerasimov, Phys. Lett. **B357**, 666 (1995).

[19] C. J. Bebek *et al.*, Phys. Rev. **D17**, 1693 (1978).

[20] H.-M. Choi and C.-R. Ji, Nucl. Phys. **A618**, 291 (1997).

[21] S. J. Brodsky and J. R. Hiller, Phys. Rev. **D46**, 2141 (1992).

[22] A. S. Bagdasaryan and S. V. Esaybegyan, Sov. J. Nucl. Phys. **42**, 278 (1985).

[23] J. S. Bell and R. Jackiw, Nuovo Cimento **A60**, 47 (1969).

[24] S. L. Adler, Phys. Rev. **177**, 2426 (1969).

[25] S. J. Brodsky, C.-R. Ji, A. Pang, and D. G. Robertson, Phys. Rev. **D57**, 245 (1998).

[26] P. L. Chung, F. Coester, and W. N. Polyzou, Phys. Lett. **B205**, 545 (1988).

[27] Z. Dziembowski, Phys. Rev. **D37**, 778 (1988).

[28] S. R. Amendolia *et al.*, Phys. Lett. **B178**, 435 (1986).

[29] CELLO coll., H.-J. Behrend *et al.*, Z. Phys **C49**, 401 (1991).

[30] CELLO coll., A. Savinov *et al.*, PHOTON 95 Conference, Sheffield (1995).

FIGURES

FIG. 1. The meson interaction potential $V_0(r)$. Our parameters are compared with the HO potential of [7], the quasi-relativistic potential of ISGW2 ($\kappa = 0.3$ and $\kappa = 0.6$) [14] and the relativistic potential of GI [1].

FIG. 2. The π^+ form factor times Q^2. We also show the predictions from a simple VMD model. The experimental data is taken from [19]

FIG. 3. The radiative $\rho^+ \to \pi^+\gamma^*$ and $\omega \to \pi^0\gamma^*$ transition form factors $F_{\rho\pi}$ and $F_{\omega\pi}$. We also show the body form factors $H_D^{\rho\pi}$ and $H_P^{\rho\pi}$

FIG. 4. The rho form factors including both Dirac and Pauli form factors on the level of CQ.

FIG. 5. The angular condition tested by $\Delta(Q^2)$. Our result is compared to other calculations [20,6]. A feature of our model is the fact that the violation of the angular condition is smaller than in other models.

FIG. 6. The $\pi^0 \to \gamma^*\gamma$ form factor $F_{\pi\gamma}$. Our result agrees with the experimental data [29,30] only for low Q^2. Using a simple power law Ansatz as suggested in [25] we get agreement for $Q^2 > 2 \text{ GeV}^2$.

FIG. 7. The π^+ form factor times Q^2 for the ground state ($1S$) and the first two radially excited states ($2S$ and $3S$).

TABLES

TABLE I. Model parameters for the best fit of the mass spectrum for both HO and linear potential for several quark masses m_q. The parameter β has been obtained from the variational principle for the ground state. For comparison, the parametrization from [7] together with the ground state masses are shown. Also shown is the pion decay constant f_π used to distinguish between the different parametrizations. All units are in GeV, except the parameter b which is in GeV^3 [GeV^2] for HO [linear] potential, f_π which is in MeV and κ which is dimensionless.

	HO	HO	HO	HO	HO	HO	linear	linear	HO[a]	linear[a]	Experiment[b]
m_q	0.150	0.180	0.190	0.200	0.250	0.300	0.150	0.250	0.250	0.220	
a	0.268	0.264	0.256	0.246	0.243	0.220	0.112	0.187	-0.144	-0.724	
b	0.0127	0.0117	0.0118	0.012	0.0092	0.0069	0.089	0.05	0.010	0.18	
κ	1.207	1.22	1.22	1.218	1.246	1.26	1.165	1.242	0.607	0.313	
β	0.4782	0.4894	0.4957	0.5019	0.5224	0.5463	0.4914	0.5277	0.3194	0.3659	
M_π	0.136	0.139	0.138	0.138	0.138	0.137	0.139	0.138	0.137	0.135	0.135…0.139
$M_{\pi(2S)}$	1.280	1.272	1.275	1.279	1.261	1.253	1.288	1.258			1.3±0.1
$M_{\pi(3S)}$	1.799	1.797	1.806	1.817	1.802	1.805	1.781	1.776			1.801±0.013
M_ρ	0.770	0.770	0.0770	0.770	0.770	0.770	0.770	0.770	0.770	0.770	0.770±0.0008
$M_{\rho(2S)}$	1.437	1.441	1.448	1.456	1.459	1.470	1.442	1.457			1.465±0.025
$M_{\rho(3S)}$	2.051	2.049	2.059	2.071	2.061	2.069	2.033	2.037			[c]
f_π	75.8	87.9	91.9	95.8	113.9	129.9	76.2	113.9	92.4	91.8	92.4±0.25

[a]Predictions from [7].
[b]Data taken from [15].
[c]See the text for a discussion of possible resonances at 1.700 ± 0.020 GeV and 2.149 ± 0.017 GeV.

TABLE II. The model parameters.

$m_q = 0.190$ GeV	$< r_D^2 >= 0.050$ fm^2
$a = 0.256$ GeV	$< r_P^2 >= 0.136$ fm^2
$b = 0.0118$ GeV^3	$\kappa_u = 0.074$
$\kappa = 1.22$	$\kappa_d = -0.048$
$\beta = 0.4957$ GeV	

TABLE III. Observables for different model parameters. The first column shows the calculations from [20,7]. The second and third columns show our predictions without and with CQ form factors.

	Choi/Ji [20,7]	our w/o CQ FF	our w/ CQ FF	Experiment
m [GeV]	0.250	0.190	0.190	
β [GeV]	0.3194	0.4957	0.4957	
$<r_D^2>$ [fm^2]	0	0	0.050	
$<r_P^2>$ [fm^2]	0	0	0.136	
κ_u	0	0	0.074	
κ_d	0	0	-0.048	
f_π [MeV]	92.4	91.9	91.9	92.4±0.025
$<r_\pi^2>$ [fm^2]	0.448	0.327	0.427	0.432±0.016
$\Gamma_{Anom}(\pi \to \gamma^*\gamma)$ [eV]	7.73	7.82	7.82	7.25±0.23
f_ρ [MeV]	151.9	231.7	153.0	152.9±3.6
$\mu_{\pi^+\rho^+}$ [GeV^{-1}]	0.782	0.603	0.701	0.741±0.038
$\mu_{\pi^0\omega}$ [GeV^{-1}]	2.35	1.81	2.27	2.33±0.06
$\rho : \mu_1$	2.2	2.27	2.63	[a]
$\rho : Q_1$	0.20	0.53	0.69	[a]

[a] There are no data available yet. However, the results of other theoretical calculations are given by: μ_1 : 2.26, 2.1, 2.3 and Q_1 : 0.37, 0.41, 0.45 from [6], [20] and [22], respectively.

TABLE IV. Shows predictions of several hadronic properties for radially excited states. $\mu_{\rho'^+\pi^+}$ and $\mu_{\omega'\pi^0}$ are transition magnetic moments for the transition from the first excited state ($2S$) to the ground state ($1S$).

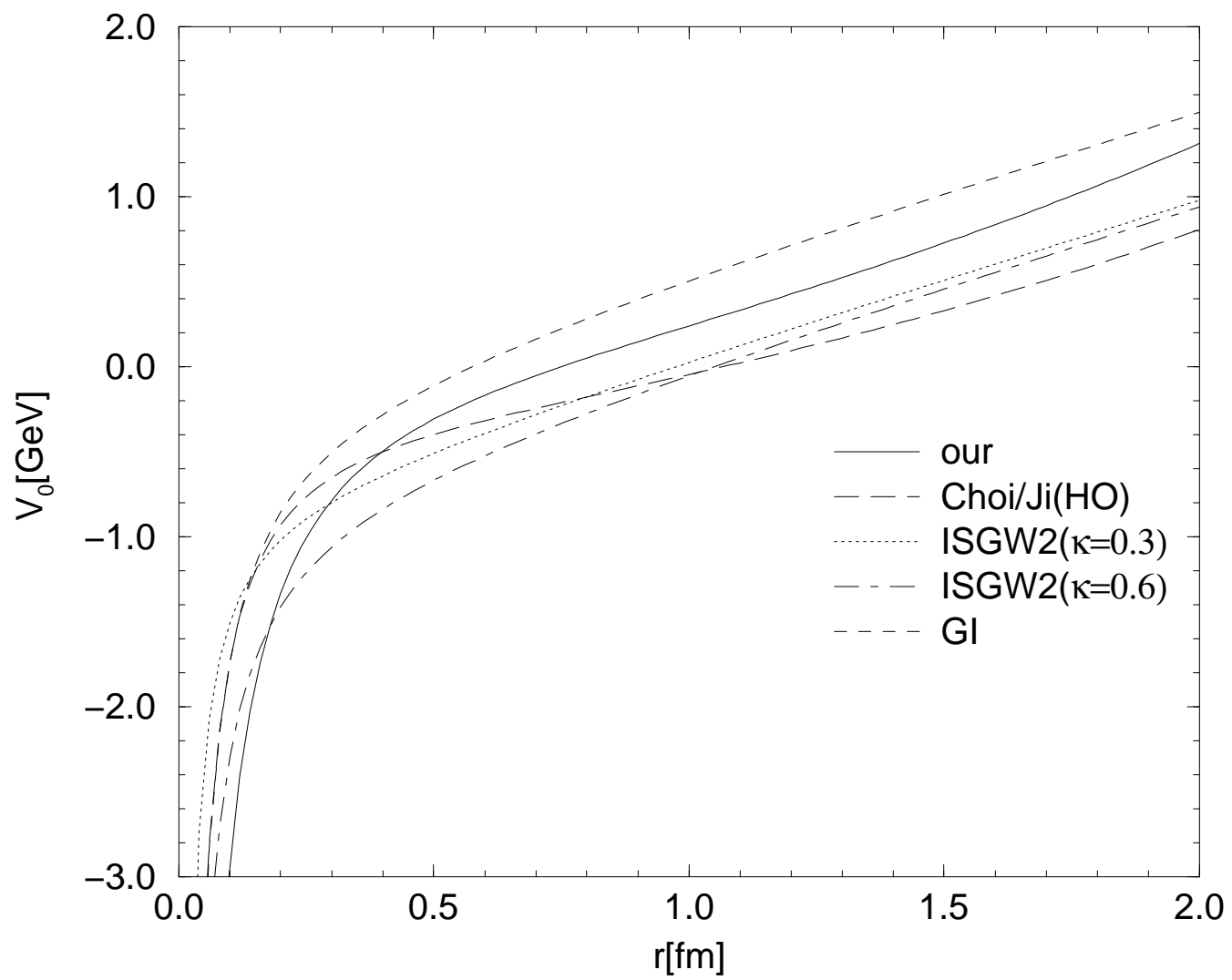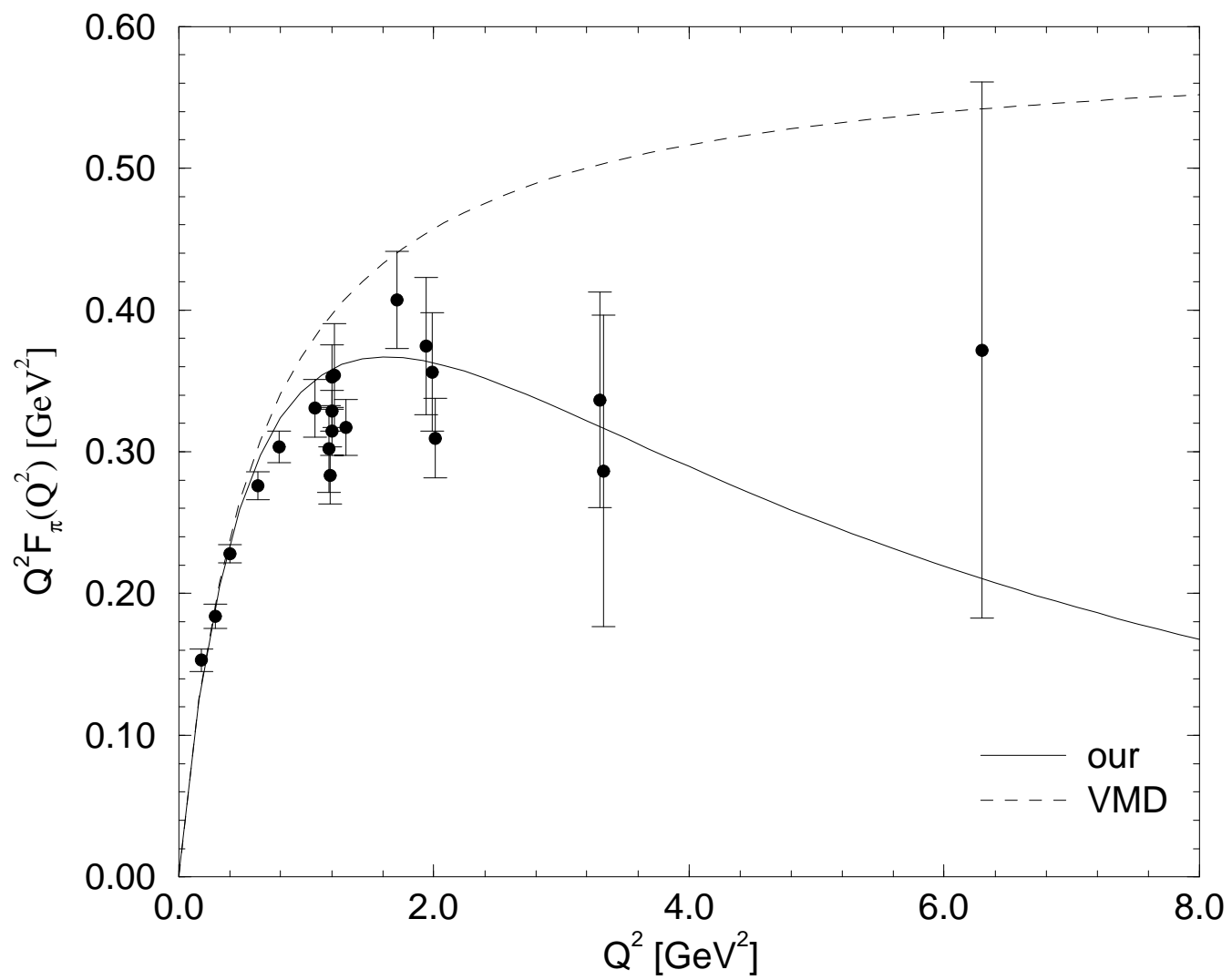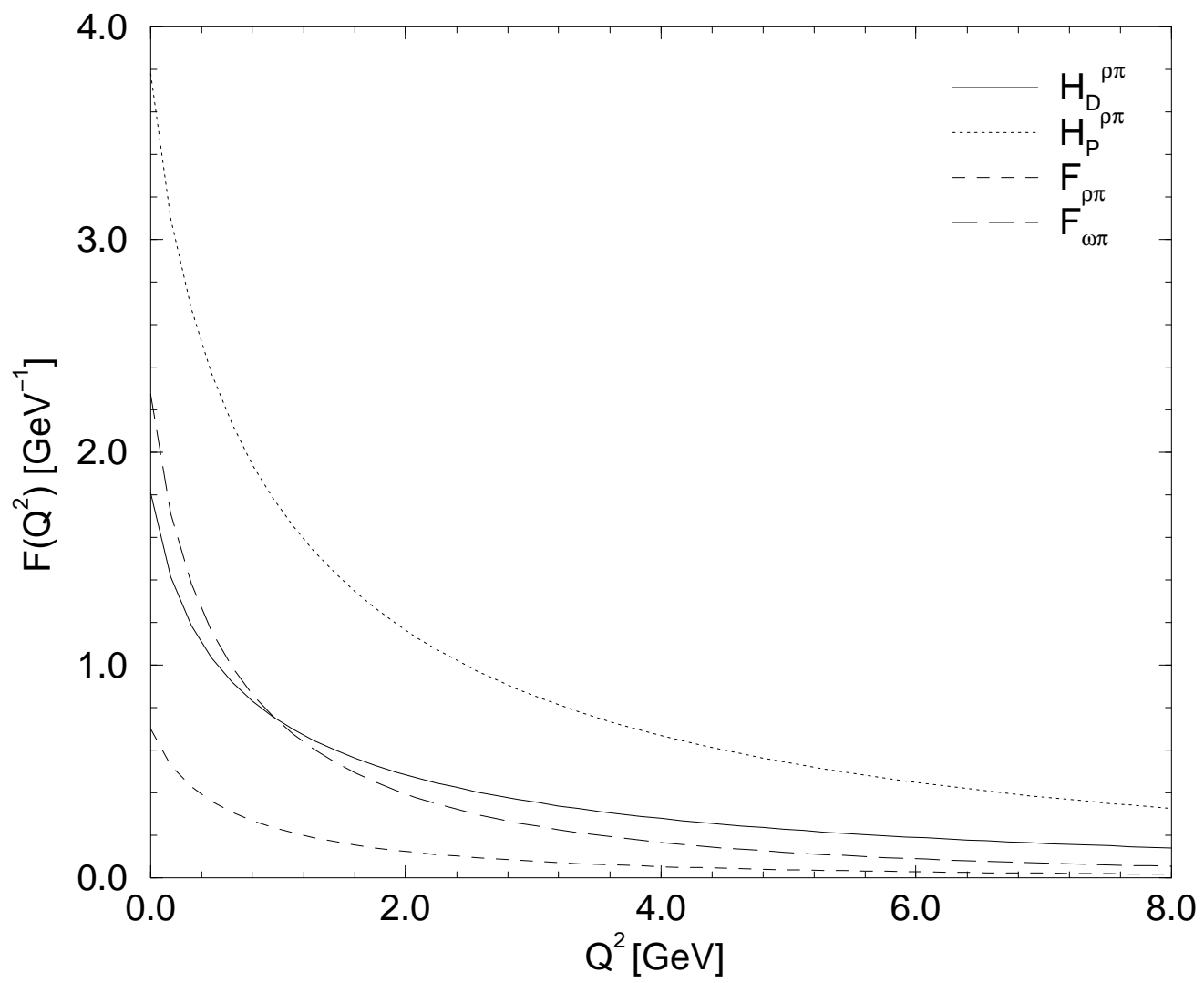|	$1S$	$2S$	$3S$
f_π [MeV]	91.9	18.6	42.3
$<r_\pi^2>$ [fm^2]	0.427	0.606	0.854
f_ρ [MeV]	153.0	90.5	96.4
$\mu_{\pi^+\rho^+}$ [GeV^{-1}]	0.70	0.55	0.48
$\mu_{\pi^0\omega}$ [GeV^{-1}]	2.27	1.79	1.58
$\mu_{\rho'^+\pi^+}$ [GeV^{-1}]	-0.23		
$\mu_{\omega'\pi^0}$ [GeV^{-1}]	-0.71		

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7